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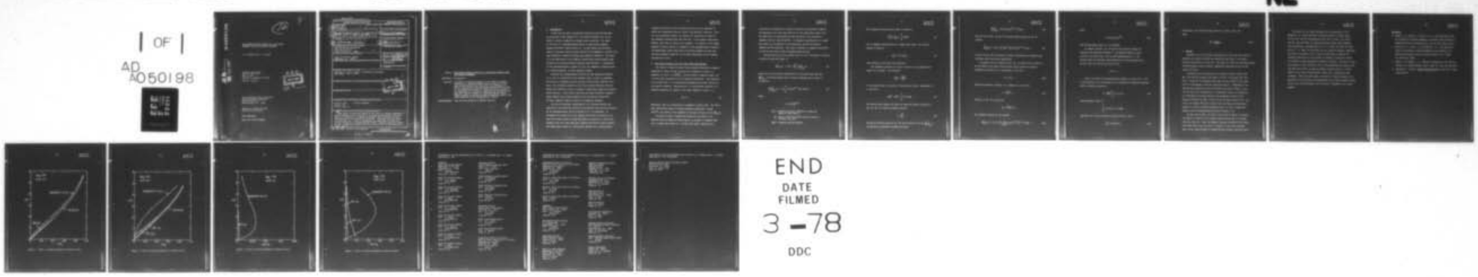

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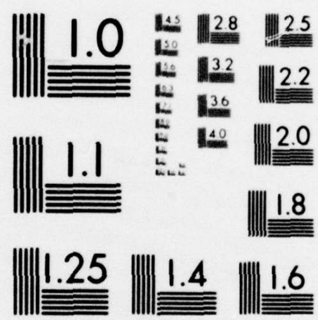

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THE LAMINAR VELOCITY PROFILE IN A FLAT PLATE BOUNDARY LAYER WITH SURFACE ROUGHNESS

G. H. Hoffman and J. L. Lumley

Technical Memorandum
File No. TM 77-150
May 11, 1977
Contract No. N00017-73-C-1418
Copy No. 27

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REPORT DOCUMENTATION PAGE

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2. GOVT ACCESSION NO.:
3. RECIPIENT'S CATALOG NUMBER:
4. TITLE (and Subtitle): 6 THE LAMINAR VELOCITY PROFILE IN A FLAT PLATE BOUNDARY LAYER WITH SURFACE ROUGHNESS.
5. TYPE OF REPORT & PERIOD COVERED: 9 Technical Memo~~randum~~.
6. PERFORMING ORG. REPORT NUMBER:
7. AUTHOR(s): 10 G. H. Hoffman ~~and~~ J. L. Lumley
8. CONTRACT OR GRANT NUMBER(s): 15 N00017-73-C-1418
9. PERFORMING ORGANIZATION NAME AND ADDRESS:
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   Post Office Box 30
   State College, PA 16801
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS:
11. CONTROLLING OFFICE NAME AND ADDRESS:
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    Washington, DC 20362
12. REPORT DATE: May 11, 1977 — 11 11 May 77
13. NUMBER OF PAGES: 14 — 12 16p.
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office):
15. SECURITY CLASS. (of this report): UNCLASSIFIED
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE:
16. DISTRIBUTION STATEMENT (of this Report):
    Approved for public release. Distribution unlimited.
    Per NAVSEA - Feb. 6, 1978.
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report):

18. SUPPLEMENTARY NOTES:
19. KEY WORDS (Continue on reverse side if necessary and identify by block number):
    boundary layer
    laminar flow
    velocity profile
    surface roughness
20. ABSTRACT (Continue on reverse side if necessary and identify by block number):

The theory of Singh and Lumley for the effect of surface roughness on a laminar boundary layer is applied to flow past a semi-infinite flat plate where the roughness is a randomly distributed small amplitude waviness characteristic of the manufacturing process. Distorted velocity profiles are calculated for a range of parameters and comparisons are made with results obtained by a phenomenological theory. Present results are nearly an order of magnitude smaller than those predicted by the phenomenological theory.

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391 007

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References: See page 10.

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Acknowledgment: This work was sponsored by NAVSEA, Code 031.

1. Introduction

In the last few years considerable theoretical work has been done by the groups at Flow Research Inc. and Physical Dynamics Inc. on the effect of surface roughness on boundary layer transition. The basis of this work is a phenomenological theory of distributed roughness advanced by Merkle, Kubota and Ko [1]. In this theory the distortion effect on the mean velocity profile caused by the roughness is treated by an eddy diffusivity concept utilizing three empirical constants. The theory is in the same spirit as the familiar zeroth-order closure methods (eddy viscosity) of treating turbulent boundary layer profiles. A consequence of this phenomenological treatment is that the roughness is characterized by a single parameter, the mean roughness height.

Recently this phenomenological theory has been applied by Kosecoff et al. [2] to a heated flat plate boundary layer in water to determine the effects of roughness and wall heating on the neutral stability curve. One of the conclusions reached is that only a modest deviation from the smooth wall (Blasius) profile is needed to drastically change the stability characteristics. Their results show that the eddy diffusivity roughness model produces quite large deviations from the Blasius profile when the mean roughness height is equal to the momentum thickness.

The aim of the present investigation is to determine whether the magnitude of the distortion effects on the boundary layer profile predicted by the phenomenological theory of Merkle et al. is reasonable. To accomplish this objective we will compare the results of Kosecoff et al. with the rational theory of Singh and Lumley [3] applied to a flat plate boundary layer with small amplitude randomly distributed surface waviness. The Singh-Lumley theory is a perturbation approach for a quite general

roughness distribution which proceeds directly from the equations of motion and consequently does not suffer from empirical constants. Since it is a perturbation theory, the results are restricted to values of the expansion parameter much less than one. In the present application, however, this does not seem to be a drawback. In contrast to the single parameter characterization of roughness by the phenomenological approach, the Singh-Lumley theory shows that the solution depends not only upon the mean roughness height but on the Taylor Microscale of the roughness distribution as well.

2. Flat Plate Boundary Layer with Small Amplitude Waviness

The theory of Singh and Lumley treats randomly distributed roughness described by $y=h(x,z)$ using a perturbation expansion in terms of a parameter ε/λ where ε is $\langle h^2\rangle_{av}^{1/2}$, the mean spatial roughness height, and λ is the Taylor microscale of the roughness distribution. The assumption is also made that h is a statistically homogeneous isotropic function of the spatial variables. Characteristic of a perturbation expansion, the expansion parameter is required to be small compared to unity, i.e.,

$$\frac{\varepsilon}{\lambda} << 1 \quad . \tag{1}$$

Physically, this is a restriction to roughness of small slope. The theory then calculates by means of matched asymptotic expansions, through $O(\varepsilon^2/\lambda^2)$, the effect of the roughness on the mean velocity profile $\langle U\rangle_{av}(y)$.

To obtain an order of magnitude estimate of the effects to be expected from the Singh and Lumley theory, we consider a roughness made up of a single wave number $\tilde{\alpha}_*$. A single wave number simplifies the

calculations considerably and since the effects of distributed roughness are expected to be of the same order as for the single wave number case, the added labor of determining the spectral density for distributed roughness does not seem worthwhile. A roughness characterized by a single wave number $\tilde{\alpha}_*$ corresponds to small amplitude waviness distributed randomly and isotropically. This type of waviness is commonly encountered in practice and results from the manufacturing process.

The mean velocity in the boundary layer in the x (streamwise) direction is given by Singh and Lumley as

$$\langle U \rangle_{av}(y) = \tilde{u}(y) + \left(\frac{\varepsilon}{\lambda}\right)^2 \left\langle W_{2_1} \right\rangle_{av}(y) \quad , \tag{2}$$

where $\tilde{u}(y)$ is the velocity distribution for the smooth wall case and $\left\langle W_{2_1} \right\rangle_{av}$ is the perturbation due to surface roughness and is found to be given by

$$\left\langle W_{2_1} \right\rangle_{av}(y) = \tilde{u}_0' \int_0^\infty (R\, \tilde{u}_0'\, \tilde{\alpha})^{1/3}\, E(\tilde{\alpha})\, \phi(s) d\tilde{\alpha} \quad , \tag{3}$$

where

$$s = y(R\, \tilde{u}_0' \tilde{\alpha})^{1/3} \quad ,$$

$\phi(s)$ = influence function tabulated by Singh and Lumley in their Table I,

u_0' = slope of the undisturbed velocity profile at the wall, y=0, and

$E(\tilde{\alpha})$ = roughness spectral density.

The roughness spectral density $E(\tilde{\alpha})$ is defined by

$$\left\langle h^2(x,z)\right\rangle_{av} = \int_0^\infty E(\tilde{\alpha})d\tilde{\alpha} \quad . \tag{4}$$

For a roughness characterized by a single wave number, the spectral density is given by

$$E(\tilde{\alpha}) = \varepsilon^2\,\delta(\tilde{\alpha}-\tilde{\alpha}_*) \quad , \tag{5}$$

where $\delta(\tilde{\alpha}-\tilde{\alpha}_*)$ is the Dirac delta function.

The expansion parameter ε/λ may be related to the roughness wave number $\tilde{\alpha}_*$ as follows: By definition

$$2\left(\frac{\varepsilon}{\lambda}\right)^2 = \overline{\left(\frac{dy}{dx}\right)^2} \quad .$$

In the present case, the surface is described by $y=h(x)$, independent of z, and hence

$$2\left(\frac{\varepsilon}{\lambda}\right)^2 = \overline{h'h'} = \int_0^\infty \tilde{\alpha}^2\,E(\tilde{\alpha})d\tilde{\alpha} \quad . \tag{6}$$

For discrete wave numbers for which the spectral density is given by Eq. (5), Eq. (6) yields the simple relation

$$\lambda = \frac{\sqrt{2}}{\tilde{\alpha}_*} \quad . \tag{7}$$

The spectral density given by Eq. (5) also allows Eq. (3) for $\left\langle W_{2_1}\right\rangle_{av}$ to be evaluated by quadrature yielding the result

$$\left\langle W_{21} \right\rangle_{av} = \tilde{u}_0' \, (R \, \tilde{u}_0' \, \tilde{\alpha}_*)^{1/3} \, \varepsilon^2 \, \phi(s_*) \quad . \tag{8}$$

With the aid of Eqs. (5) and (7) the mean velocity given by Eq. (2) becomes

$$\left\langle U \right\rangle_{av}(y) = \tilde{u}(y) + \frac{1}{2} \, (\varepsilon\tilde{\alpha}_*)^2 \, \tilde{u}_0'(R \, \tilde{u}_0' \, \tilde{\alpha}_*)^{1/3} \, \varepsilon^2 \, \phi(s_*) \quad . \tag{9}$$

We note that Eq. (9) is written in terms of dimensionless variables with reference quantities as yet unspecified.

To compare with the results of Ref. (2), the basic flow is chosen to be the Blasius flat plate solution which can be written in terms of similarity variables as, cf. Rosenhead, p. 222, [4],

$$\tilde{u} = f'(\eta) \quad , \tag{10}$$

where the similarity variable η is related to x, y and R by

$$\eta = \sqrt{R/2x} \;\; y \quad . \tag{11}$$

Then $\tilde{u}_0'$ in Eq. (9) is given by

$$\tilde{u}_0' = \sqrt{R/2x} \; f''_0 \quad .$$

For a Blasius profile Eq. (9) becomes

$$\left\langle U \right\rangle_{av}(\eta) = f'(\eta) + \frac{1}{2} \, \frac{R}{(2x)^{2/3}} \, (f_0'')^{4/3} \, \tilde{\alpha}_*^{7/3} \, \varepsilon^4 \, \phi(K\eta) \quad , \tag{12}$$

where

$$K = (2x\, f_0''\, \tilde{\alpha}_*)^{1/3} \quad , \tag{13}$$

and from Rosenhead, Table V.1, f_0''=0.46900.

To compare with Ref. (2), we take as the reference length the boundary layer momentum thickness θ. The Reynolds number is based on θ and y as given by Eq. (11) is made dimensionless by θ. The present theory is therefore characterized by the three parameters R_θ, $\tilde{\alpha}_*$ and ε plus the restriction that the the

$$\varepsilon\, \tilde{\alpha}_* << 1 \quad .$$

Since θ is taken as the characteristic length, x in Eqs. (11) - (13) is not arbitrary but is determined by the definition of momentum thickness which in Blasius variables is

$$\theta = \sqrt{2x/R} \int_0^\infty f'(1-f')d\eta \quad . \tag{14}$$

From Rosenhead, Table V.1

$$\int_0^\infty f'(1-f')d\eta = 0.46960 \quad ,$$

and hence the relation between x/θ and R_θ from Eq. (14) is

$$\frac{x}{\theta} = 2.267325\, R_\theta \quad . \tag{15}$$

Furthermore, the relation between y/θ and η, from Eq. (11), now reads

$$\frac{y}{\theta} = \frac{\eta}{0.46960} \quad . \tag{16}$$

3. Results

Kosecoff et al. [2] in their calculations of flat plate mean velocity profiles, the results of which are plotted in their Fig. 1, use values of R_θ=175 and ε/θ=0.3, 1.0 and 2.0. The curve for ε/θ=0.3 is indistinguishable from the Blasius profile, however, significant departures occur for the cases ε/θ=1 and 2.

Calculations were performed for the Singh and Lumley theory using Eqs. (12) and (13) together with Table I of Ref. 3 for R_θ=175 and ε/θ=1 and 2. Values of the roughness wave number $\tilde{\alpha}_*$ were chosen in the range 0.05 to 0.20 keeping in mind the restriction $\varepsilon\tilde{\alpha}_* \ll 1$. Comparison of the mean velocity profiles for the two theories are shown in Figs. (1) and (2) for ε/θ=1 and 2 respectively. To show more clearly the differences in predictions of the two theories, the disturbance velocity ratio $\delta u/u_e$ is presented in Figs. (3) and (4) for the same values of ε/θ. The latter two figures show quite clearly that the Singh and Lumley theory predicts a velocity profile disturbance nearly an order of magnitude smaller than the phenomenological theory of Merkle et al.

We note that in Figs. (2) and (4) the curves for $\tilde{\alpha}_*$=0.1 are beyond the limits of validity of the present theory because $\varepsilon\tilde{\alpha}_*$ is no longer much less than one. Also, the negative velocity predicted by the Singh and Lumley theory near the wall is evidence of the theory breaking down in this region because of linearizing the boundary condition there.

The reason for the large difference in the predictions of the Kosekoff et al. theory and that of Singh and Lumley is almost surely not the fact that one is *ad hoc* while the other is exact, but rather the fact that the former is calibrated against typical sand roughness drag coefficients, corresponding to $\epsilon/\lambda >> 1$, while the latter is valid for $\epsilon/\lambda << 1$. Hence, the theories are not, properly speaking, comparable. The theory of Kosekoff et al., however, particularly as regards its stability predictions in the presence of heating, is being put forward as relevant to small axisymmetric underwater vehicles. The type of roughness present on such vehicles is due to the manufacturing processes used, and since every effort is made to produce a smooth finish, it is most likely to be a small-slope waviness, rather than a large-slope sand roughness. Hence, we would expect the theory of Singh and Lumley to be applicable to these bodies, and consequently that a given roughness height would be far less detrimental than the theory of Kosekoff et al. would suggest.

References

1. Merkle, C. L., Kubota, T. and Ko, D. R. S., "An Analytical Study of the Effects of Surface Roughness on Boundary Layer Transition," AFOSR-TR-75-0190, Flow Research Rept. No. 40, Kent, Wash. (1974).
2. Kosecoff, M. A., Ko, D. R. S. and Merkle, C. L., "An Analytical Study of the Effect of Surface Roughness on the Stability of a Heated Water Boundary Layer," Physical Dynamics, Inc., Report No. PDT-76-131 (1976).
3. Singh, K. and Lumley, J. L., "Effects of Roughness on the Velocity Profile of a Laminar Boundary Layer," Appl. Sci. Res. 24, 168 (1971).
4. Rosenhead, L. (editor), Laminar Boundary Layers, Oxford Univ. Press, London (1963).

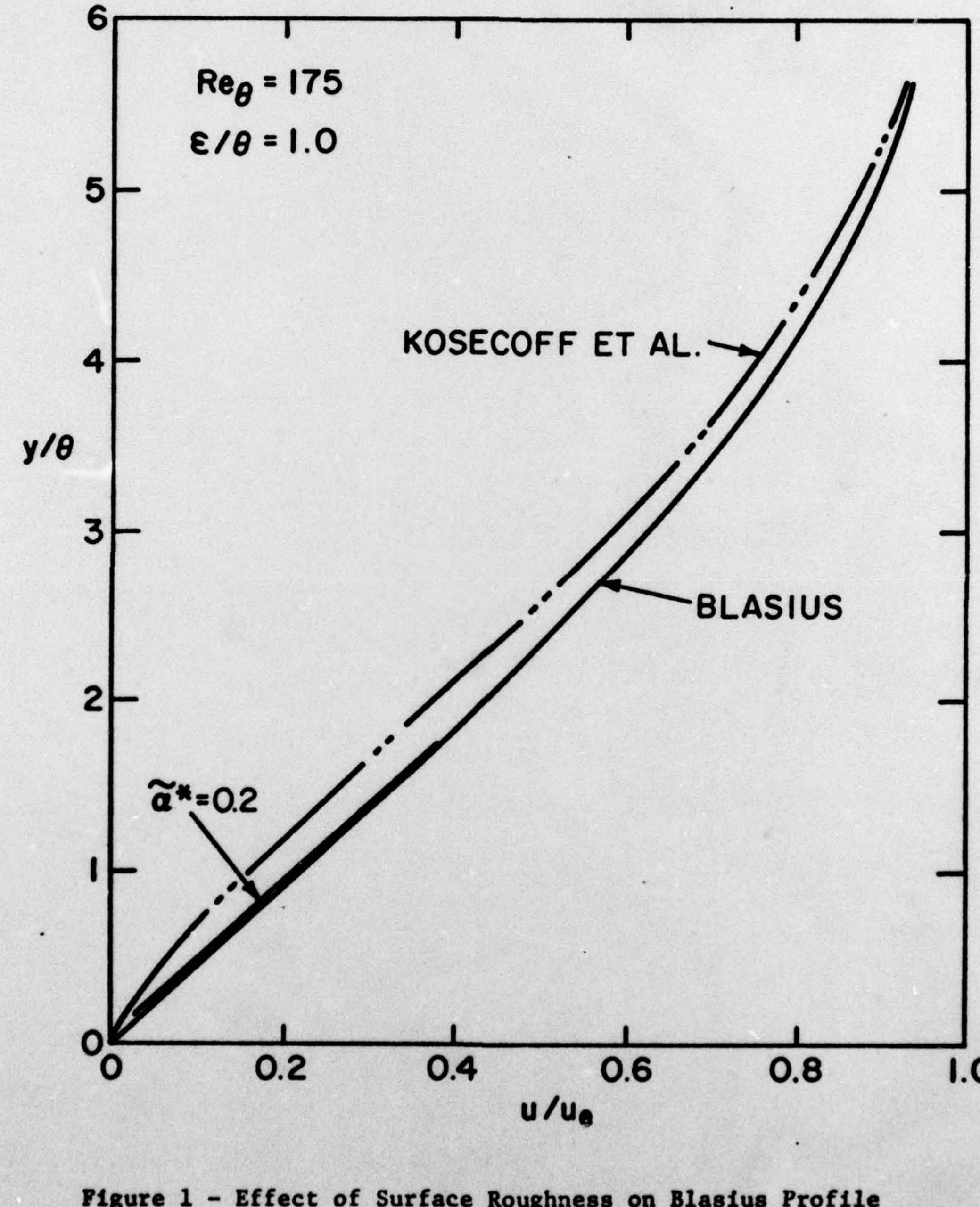

Figure 1 - Effect of Surface Roughness on Blasius Profile

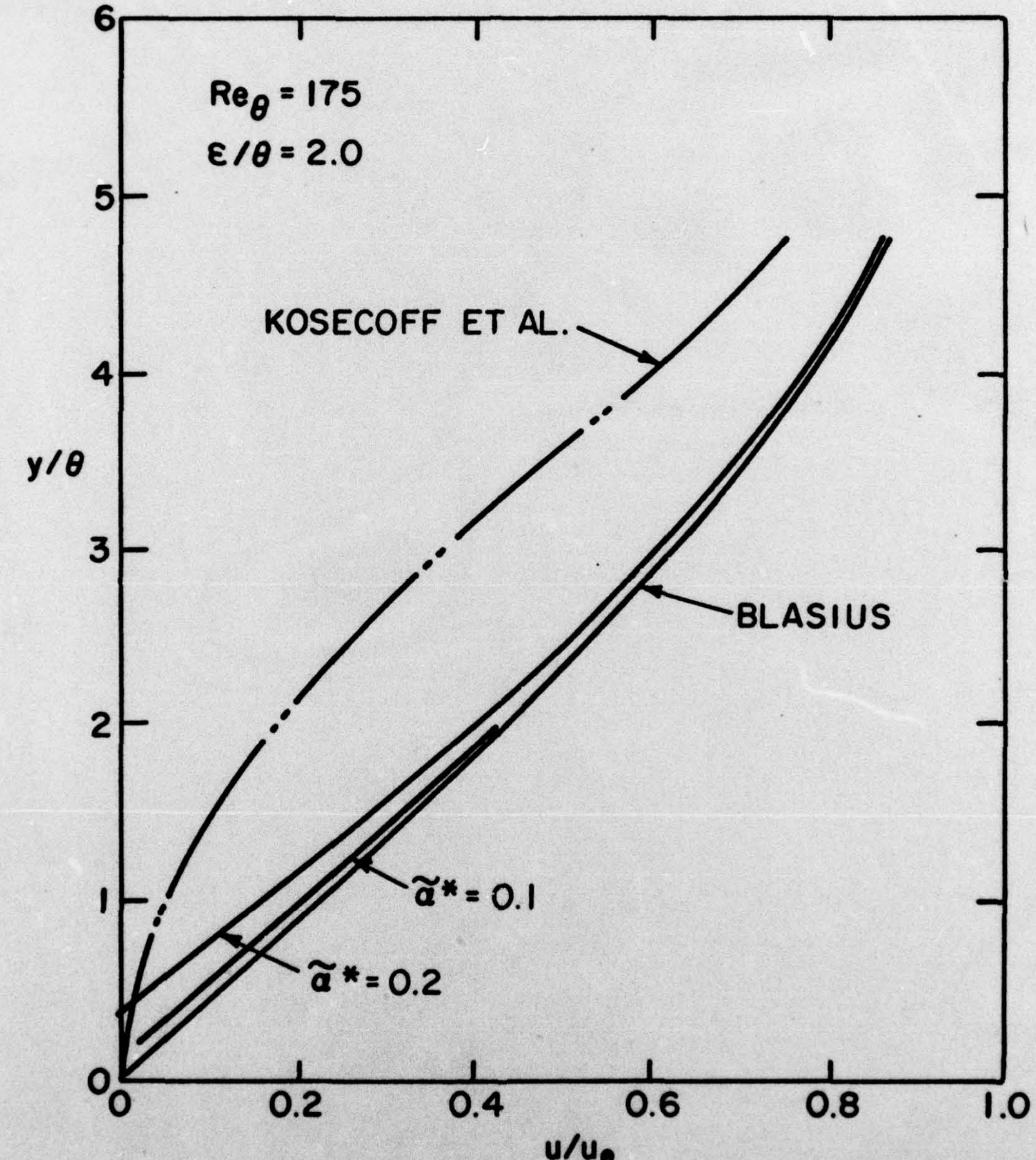

Figure 2 - Effect of Surface Roughness on Blasius Profile

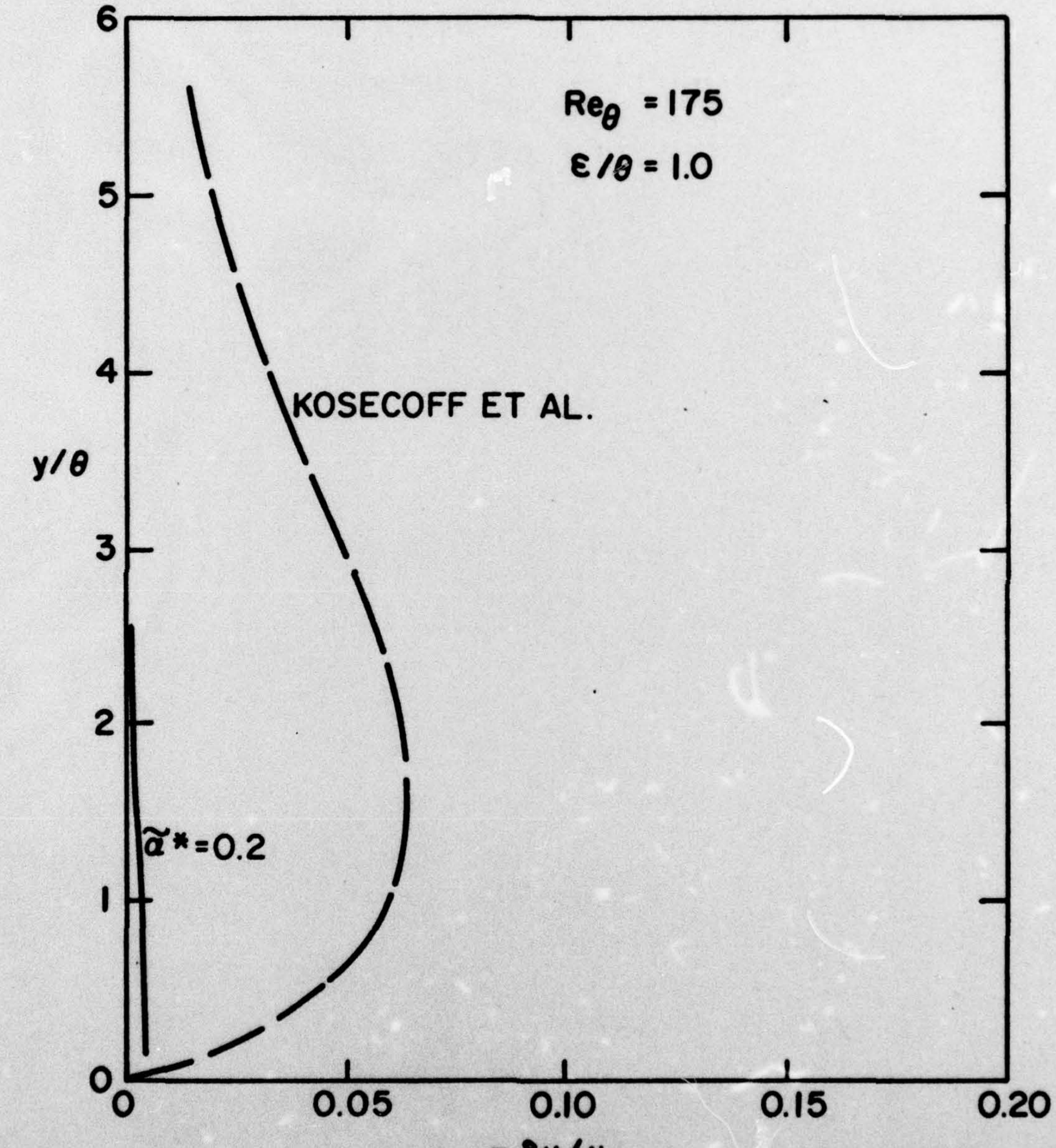

Figure 3 - Effect of Surface Roughness on Blasius Profile

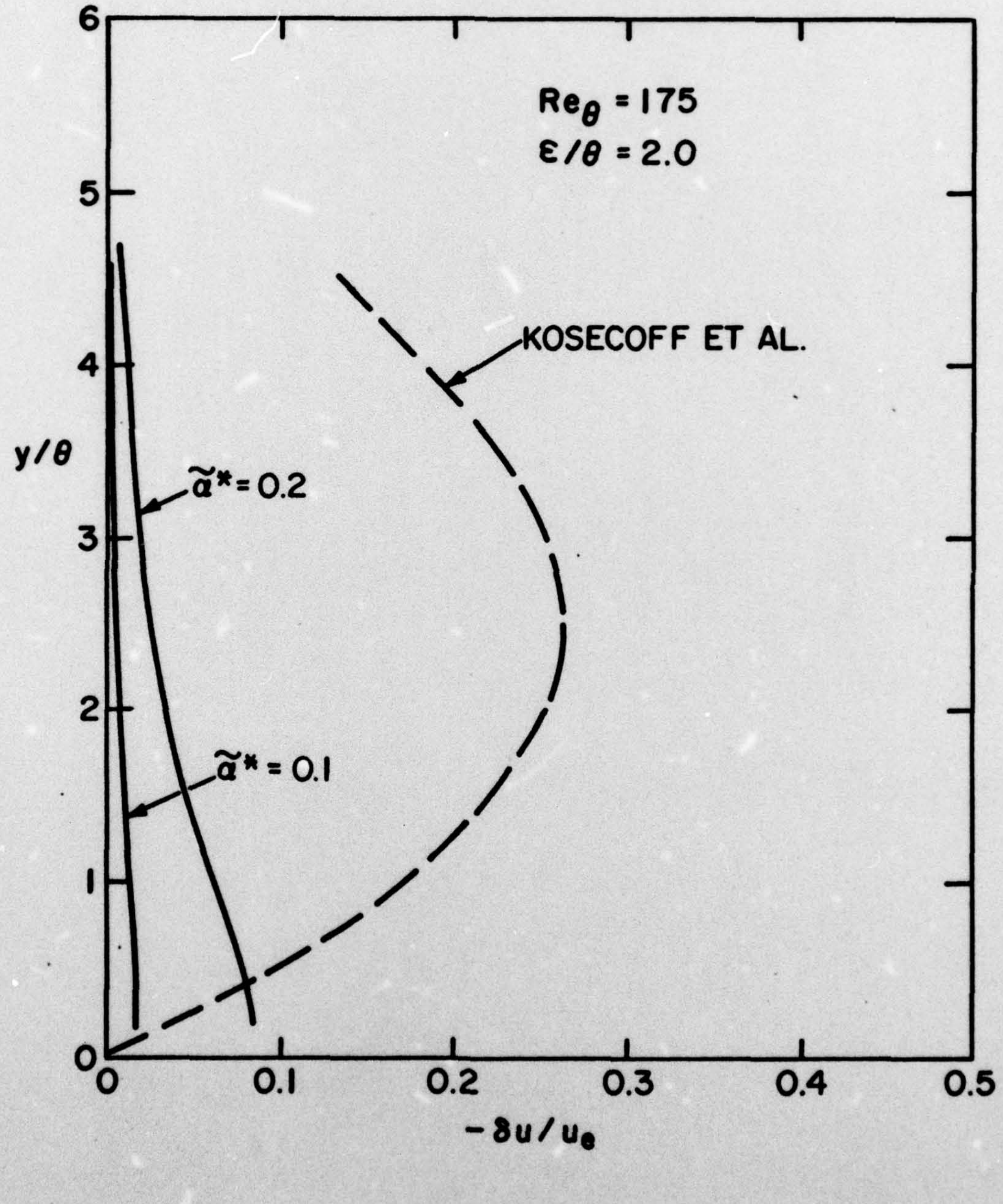

Figure 4 - Effect of Surface Roughness on Blasius Profile

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